Exponential Decay

Radioactive decay:

Initial number of nuclei is N₀

Rate of decay = $-\lambda N \checkmark$

This is the *activity* of the nuclei.

The same proportion of nuclei decay in the same period of time \checkmark

This is the same as saying that the probability of a nucleus decaying is λ , where

 $\frac{dN}{dt} = -\lambda N \qquad \lambda \text{ is the decay constant } \checkmark$ there is a '-' sign because the number is falling \checkmark

The solution of this differential equation is:

 $N = N_0 e^{-\lambda t}$ from this, we can use the log relationship to obtain

 $\ln N - \ln N_0 = -\lambda t \checkmark$

Half-life

When the number of nuclei has dropped to half, N = 0.5 N₀ \checkmark

The half-life, $t_{1/2} = \frac{0.693}{\lambda}$

Eg There were 100 dice in the experiment. The probability of a 'decay' was $1/6 \checkmark$

 $\lambda = 1/6$ \checkmark

- a) What is the predicted 'half-life', $t_{1/2}$? $t_{1/2} = 0.693 \div 1/6 = 4.2$ throws \checkmark
- b) How many throws are likely to be needed for 20 dice to remain?

 $\ln(0.2) = -1.61 \implies$ no of throws = $-1.61 \div 1/6 = 9.66$ (i.e. between 9 and 10) \checkmark

c) If there were 10 000 dice to start with, how many dice would be likely to remain after 10 throws?

 $N = 10\ 000 \times e^{-1/6 \times 10} = 1900$ \checkmark

Straight Line Graph

The solution of $N = N_0 e^{-\lambda t}$ is

 $\ln N - \ln N_0 = -\lambda t \quad \text{or} \quad \ln N = -\lambda t + \ln N_0$

This will give a straight line graph:

y = mx + c

the y axis will be $\ln N \checkmark$

the *x* axis will be $t \checkmark$

the value of the y-intercept will be $\ln N_0 \checkmark$

the value of the gradient will be - λ \checkmark

